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Baryon Regge Trajectories in the Light of the $1/N_c$ Expansion

J. L. Goity [a,b], N. Matagne [b,c]

[a] *Department of Physics, Hampton University, Hampton, VA 23668, USA*
[b] *Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA*
[c] *University of Liège, Institute of Physics B5, Sart Tilman, B-4000 Liège 1, Belgium*

Abstract

We analyze Regge trajectories in terms of the $1/N_c$ expansion of QCD. Neglecting spin-orbit contributions to the large N_c baryon mass operator, we consider the evolution of the spin-flavor singlet component of the masses with respect to the angular momentum. We find two distinct and remarkably linear Regge trajectories for symmetric and for mixed symmetric spin-flavor multiplets.

1. Introduction

The ordering of hadronic states on approximately linear Regge trajectories in the Chew-Frautschi plot is one of the most remarkable features of the QCD spectrum. It manifests the underlying non-perturbative QCD dynamics, which at long distances becomes dominated by the string-like behavior that leads to confinement. In fact this picture has been the motivation for the development of string/flux tube models of hadrons [1], which contemporarily are described as effective theories in the so called AdS/QCD framework [2]. The latter is valid in the large N_c limit, N_c being the number of colors, and has been applied almost exclusively to mesons, while extensions to baryons are being explored [3,4]. Furthemore, it has been shown recently that flux tube model and large N_c mass formulas are compatible [5]. Regge trajectories have also been recently considered in the context of the quark-diquark picture of baryons [6].

In this work we will analyze the baryon Regge trajectories in the light of the $1/N_c$ expansion, which is in principle an approach consistent with QCD. The $1/N_c$ expansion for baryons is based on the emergent $SU(6)$ spin-flavor symmetry (for three light flavors) in the large N_c limit [7,8,9]. For excited baryons, the usual approach consists in organizing states into multiplets of the $SU(6) \times O(3)$ group. Even if it has been shown that, for mixed symmetric multiplets, this symmetry is broken at order $\mathcal{O}(N_c^0)$ by spin-orbit interactions, it is a phenomenological fact that these interactions are very small (in the real world with $N_c = 3$ they have a magnitude expected for $\mathcal{O}(N_c^{-2})$ effects). Thanks to this observation, the usage of the $SU(6) \times O(3)$ symmetry at leading order is justified. Following this approach, various works [10,11,12,13,14,15,16] have shown that the $1/N_c$ expansion is a very useful tool for analyzing the baryon spectrum. In this work, we assume that the magnitude of spin-orbit interactions is small for highly excited states, *e.g.* for states belonging to $[\mathbf{70}, 5^-]$ and $[\mathbf{56}, 6^+]$ multiplets. Indeed, because of a lack of data, it is not possible to make a detailed study of these multiplets as it was done in Refs. [11,12,13,14,15,16]

Email addresses: goity@jlab.org (J. L. Goity), nmatagne@ulg.ac.be (N. Matagne).

for lower excitations.

In the $1/N_c$ expansion, the mass operator for a given $SU(6)\times O(3)$ multiplet is expressed in terms of a series in effective operators [10,11,12,13,14,15,16] ordered in powers of $1/N_c$. The coefficients associated with the operators are obtained by fitting to the empirical masses. The various analyses have shown that these coefficients are of natural magnitude or smaller (dynamically suppressed), lending support to the consistency of the framework. To a first approximation, it turns out that the main features of the spectrum can be captured by taking into account a few operators, namely the $\mathcal{O}(N_c)$ spin-flavor singlet operator, one $\mathcal{O}(1/N_c)$ hyperfine operator, and the strangeness operator of $\mathcal{O}(N_c^0 m_s)$. For a few multiplets, the hyperfine $SU(3)$ breaking $\mathcal{O}(m_s/N_c)$ operator $\hat{S}\cdot\hat{G}_8 - \frac{1}{2\sqrt{3}}\,\hat{S}^2$ ($\hat{G}_8$ denotes the eighth component of the axial current, which is one of the $SU(6)$ spin-flavor generators) is necessary for achieving a consistent fit to the empirical masses. For the finer aspects of the spectrum, more operators are of course needed. The coefficients of the operators considered in this work are $\mathcal{O}(N_c^0)$, and for $SU(3)$ singlet operators the coefficients differ from multiplet to multiplet by amounts $\mathcal{O}(1/N_c)$. The purpose of this work is to analyze the evolution of the coefficients as a function of the $O(3)$ quantum number ℓ. In particular we focus on the evolution of the coefficient associated with the leading spin-flavor singlet operator, which determines the Regge trajectories.

2. Analysis

We start by considering the $[\mathbf{56},\ell]$ and the $[\mathbf{70},\ell]$ multiplets of $SU(6)\times O(3)$, which correspond respectively to the symmetric (S) and mixed-symmetric (MS) spin-flavor multiplets at $N_c = 3$. We entirely disregard possible mixings between these multiplets [17], an approximation that seems to be consistent phenomenologically as shown by analyses of strong transition amplitudes [18] as well as electromagnetic transitions [19].

For the ground state baryons, which consist of the octet and decuplet in the $[\mathbf{56},0^+]$ multiplet, the mass formula reads:

$$\hat{M}_{\rm GS} = N_c\, c_1 \mathbb{1} + \frac{1}{N_c} c_{\rm HF}\left(\hat{S}^2 - \frac{3}{4}N_c\right) - c_{\mathcal{S}}\,\hat{\mathcal{S}} + \frac{1}{N_c}\,c_4\left(\hat{I}^2 - \hat{S}^2 - \frac{1}{4}\hat{\mathcal{S}}^2\right), \qquad (1)$$

where $\hat{S}$, $\hat{I}$ are the baryon spin and isospin operators respectively and $\hat{\mathcal{S}}$ is the strangeness operator. The hyperfine term has been defined such that in the limit of a non-relativistic quark picture it corresponds to the operator $\frac{1}{N_c}\sum_{i\neq j}\vec{s}_i\cdot\vec{s}_j$, *i.e.* with the one-body pieces removed. The hyperfine $SU(3)$ breaking operator, mentioned in the introduction, has been defined in a such way that it does not contain terms linear in the strangeness operator $\hat{\mathcal{S}}$, and clearly does not contribute to the masses of non-strange ground state baryons.

For excited baryons with $\ell > 0$, the hyperfine interaction of interest can be defined following the large N_c Hartree picture of the baryon [20]: an excited quark carrying the orbital angular momentum and a core made out of the rest $N_c - 1$ quarks sitting in the ground state (for $N_c = 3$ one can identify the core with a diquark). This motivates the choice of hyperfine operator as the one that takes into account the hyperfine interactions between core quarks only. A second hyperfine operator involves the interaction between core quarks and the excited quark. In MS states one can separate these two hyperfine interactions explicitly; it was shown that the latter hyperfine effect is much weaker, and thus we neglect it here. Therefore, for excited baryons, except the $[\mathbf{56},2^+]$ multiplet, we use the following form for the mass operator:

$$\hat{M}' = N_c\, c_1 \mathbb{1} + \frac{c_{\rm HF}}{N_c}\left(\hat{S}^{c\,2} - \frac{3}{4}(N_c-1)\mathbb{1}\right) - c_{\mathcal{S}}\,\hat{\mathcal{S}}, \quad (2)$$

where $\hat{S}^c$ is the spin operator of the core. Note that the mass formulas generalize beyond the quark model, as they are entirely given in terms of generators of the spin-flavor group, and thus, only the spin-flavor nature of the states will matter.

For the $[\mathbf{56},2^+]$, we add to the mass operator the contribution of the hyperfine $SU(3)$ breaking operator, which we have modified to be expressed in terms of core operators and to have no term linear in the strangeness of the core:

$$\hat{M}' = N_c\, c_1 \mathbb{1} + \frac{c_{\rm HF}}{N_c}\left(\hat{S}^{c\,2} - \frac{3}{4}(N_c-1)\mathbb{1}\right) - c_{\mathcal{S}}\,\hat{\mathcal{S}} + \frac{4\,c_4}{3\,N_c}\left(\sqrt{3}\,\hat{S}^c\cdot\hat{G}_8^c - \frac{1}{2}\,\hat{S}^{c\,2} - \frac{1}{8}\,N_c\,\hat{\mathcal{S}}^c\right). \quad (3)$$

For non-strange excited baryons, the matrix elements of the mass operators in the different cases are as follows:

$$M'_{\rm S}(S) = N_c\, c_1 + \frac{N_c-2}{N_c^2}c_{\rm HF}\left(S(S+1) - \frac{3}{4}N_c\right),$$

$$M'_{\rm MS}(S=I) = N_c\, c_1 + \frac{c_{\rm HF}}{N_c} \times \left(\frac{N_c+2}{N_c} S(S+1) - \frac{3}{4} N_c + \frac{1}{2} \right),$$

$$M'_{\rm MS}(S=I-1) = N_c\, c_1 + \frac{c_{\rm HF}}{N_c} \left(S(S+2) - \frac{3}{4}(N_c-2) \right),$$

$$M'_{\rm MS}(S=I+1) = N_c\, c_1 + \frac{c_{\rm HF}}{N_c} \left(S^2 - \frac{3}{4} N_c + \frac{1}{2} \right). \qquad (4)$$

For $N_c = 3$ the mass formulas become:

$$N_{\rm GS} = 3\,c_1 - \frac{1}{2}\,c_{\rm HF},\ \Delta_{\rm GS} = 3\,c_1 + \frac{1}{2}\,c_{\rm HF},$$

$$N_{\rm S} = 3\,c_1 - \frac{1}{6}\,c_{\rm HF},\ \Delta_{\rm S} = 3\,c_1 + \frac{1}{6}\,c_{\rm HF},$$

$$N_{\rm MS}\left(S=\frac{1}{2}\right) = 3\,c_1 - \frac{1}{6}\,c_{\rm HF}, \qquad (5)$$

$$N_{\rm MS}\left(S=\frac{3}{2}\right) = \Delta_{\rm MS}\left(S=\frac{1}{2}\right) = 3\,c_1 + \frac{1}{6}\,c_{\rm HF},$$

where we denote $N \equiv M_N$, etc. Note that for the MS states we need to specify the total quark spin S. The case of strange baryons where we neglect the $SU(3)$ breaking hyperfine interaction is obvious, except for the $SU(3)$ singlet Λ states in the **70**-plets, where the mass formula becomes:

$$\Lambda^1_{\rm MS} = 3\,c_1 - \frac{1}{2}\,c_{\rm HF} + c_{\mathcal{S}}. \qquad (6)$$

For the $[\mathbf{56}, 2^+]$, the matrix elements of the $SU(3)$ breaking hyperfine operator are lengthy to calculate, and we direct the reader to Refs. [16,21] for details.

The coefficients c_1, $c_{\rm HF}$, $c_{\mathcal{S}}$ and c_4 are determined by fitting to the masses of the corresponding multiplet. Tables 1 and 2, for **56**- and **70**-plets baryons respectively, display the baryons listed by the Particle Data Group [22] along with their masses. Some of them ($\ell \leq 4$) can be identified with a good level of confidence as belonging to a definite $SU(6) \times O(3)$ multiplet. For the highest excitations ($\ell = 5, 6$), the situation is less clear and the identifications proposed are based on Ref. [23]. The Tables also display the results for the coefficients c_1, $c_{\rm HF}$, $c_{\mathcal{S}}$ and c_4, and the theoretical masses resulting from the fits. We note here that in the MS states there are two mixing angles, which correspond to the mixing of the octet states with quark spin $S = \frac{1}{2}$ and $\frac{3}{2}$. In the fit, these mixings are disregarded because they only originate through the presence of mass operators we have neglected. We have checked that this approximation does not affect in any significant way the conclusions of this work.

In the case of the GS baryons, as already announced above, the hyperfine $SU(3)$ breaking operator has to be include in the analysis because it affects the determination of $c_{\rm HF}$ through the fit. The result for $c_{\rm HF}$ is then consistent with the value obtained from the N-Δ mass splitting. The χ^2 is still large because of the $SU(3)$ sub-leading terms that have been disregarded. The inclusion of the higher order terms shows the improvement expected in the $1/N_c$ expansion [24]. The situation is similar in the $[\mathbf{56}, 2^+]$ multiplet, where the hyperfine $SU(3)$ breaking operator has to be included in order to have a consistent fit. One criterion for this consistency is that the values of the coefficients c_1, $c_{\rm HF}$ and $c_{\mathcal{S}}$ are in agreement with the corresponding values obtained in the analysis that includes a complete basis of operators [13].

In the $[\mathbf{70}, 1^-]$ multiplet, the large χ^2 is primarily due to the exclusion of the spin-orbit operator. That operator produces the splitting between the $SU(3)$ singlet Λ states, and the failure to describe that splitting gives the main contribution to the χ^2. This has virtually no effect on the issues we analyze here. For the **70**-plets we do not need to include the hyperfine $SU(3)$ breaking term. Note that the available information about the $[\mathbf{56}, \ell = 4, 6]$ and the $[\mathbf{70}, \ell = 2, 3, 5]$ states is somewhat limited. In each case, the information available is sufficient for determining the coefficient c_1 with enough accuracy for the purpose of this work, but the hyperfine and strangeness splittings can be only roughly determined.

The main focus of our study is the relation across multiplets of the leading order coefficient c_1. Figure 1 shows the plot $(N_c\, c_1)^2$ vs ℓ. It displays two distinct Regge trajectories corresponding to the $[\mathbf{56}, \ell]$ and the $[\mathbf{70}, \ell]$ states. In the Hartree picture, the splitting between S and MS trajectories is due to the exchange interaction between the excited quark and the core. Indeed, this exchange interaction turns out to be different for S and MS representations, being order N_c^0 in the first case and order $1/N_c$ in the second case. This implies that in large N_c limit there should be two distinct trajectories. The linear fits to the trajectories in units of GeV^2 are as follows [1] :

[1] We considered a fit with a single trajectory, which gives $\chi^2_{\rm dof} = 7.68$, to be compared to the values 0.57 and 0.06 for the fits to the S and MS trajectories respectively.

$$(3\,c_1([\mathbf{56},\ell]))^2 = (1.179 \pm 0.003) + (1.05 \pm 0.01)\,\ell,$$
$$(3\,c_1([\mathbf{70},\ell]))^2 = (1.34 \pm 0.02) + (1.18 \pm 0.02)\,\ell. \quad (7)$$

We note that the results for c_1 obtained with only non-strange baryons agree, as one would expect, with those obtained including the strange ones. It is remarkable that the spin-flavor singlet piece of the squared masses fit so well on linear Regge trajectories. The spread observed in the Regge trajectories given in terms of the physical masses is, therefore, due to the non-singlet spin-flavor components of the masses, which are dominated by the hyperfine components. For the splitting between **56**- and **70**-plet, the following linear relation gives a fair approximation:

$$(c_1([\mathbf{56},\ell]) - c_1([\mathbf{70},\ell]))^2 = (5.3 + 4.4\,\ell) \times 10^{-4}\,\mathrm{GeV}^2. \quad (8)$$

This corresponds to a mass splitting that increases with ℓ, going from ~ 70 MeV at the $\ell = 0$ intersect to ~ 170 MeV at $\ell = 6$. Since hyperfine terms have this magnitude or larger, the differentiation of the two trajectories can only be clearly seen upon removal of those terms as we have done here. One can notice that the identification of the resonance N(2600) as belonging to the $[\mathbf{70}, 5^-]$ multiplet is well supported by our study. The situation for the N(2700) and $\Delta(2950)$ remains however open.

Note that the quantity with $\mathcal{O}(N_c^0)$ slope is $N_c c_1^2$ rather than the one we plotted. It is, therefore, somewhat of a coincidence that at $N_c = 3$ the Regge slopes of mesons and of $N_c^2 c_1^2$ are so similar. Furthemore, in large N_c limit a plot linear or quadratic in c_1 would be equivalent, the reason being that the baryon masses are order N_c while the splittings between multiplets are order N_c^0. In the real world, they differ slightly, with the quadratic plot giving the best approximation to linear trajectories.

Taking into account the different definition of the hyperfine operator used in this work, which affects the values of c_1, we have verified that our results for c_1 correspond to those obtained in the analysis Refs. [11,12,13,14,15,16] where complete bases of operators are used. This is a consistency check on the irrelevance of the operators we have neglected for the purpose of our analysis. A similar comment applies to the other coefficients c_{HF}, $c_{\mathcal{S}}$ and c_4 [2].

It is interesting to notice that the strength of the HF interaction tends to increase with ℓ. This is shown clearly by the $[\mathbf{70}, 1^-]$ and the $[\mathbf{56}, 2^+]$ multiplets, where the strength is significantly larger than for the GS baryons. Unfortunately, for baryons with $\ell > 2$, c_{HF} has large uncertainty and we cannot establish that trend. According to the $1/N_c$ expansion, the value of c_{HF} differs by $\mathcal{O}(1/N_c)$ across multiplets, but in reality it changes by a factor larger than two in going from the GS to the $\ell = 2$ baryons. This can be explained by the fact that the hyperfine interaction is more sensitive to the effective size of the core than the other terms in the mass formulas. In particular, in the quark-diquark picture of the baryon, this sensitivity in the hyperfine effect indicates a reduction in the size of the diquark that is significant. The strangeness coefficient $c_{\mathcal{S}}$ seems to be bigger for the ground state and the $[\mathbf{56}, 2^+]$ multiplet than for the other cases. We note that the inclusion of the hyperfine $SU(3)$ breaking leads to an enhancement of the fit value of $c_{\mathcal{S}}$. In the $[\mathbf{70}, 1^-]$, a more detailed analysis, including an additional $SU(3)$ breaking spin singlet operator [12], leads to an enhancement of $c_{\mathcal{S}}$ as well, bringing it more in line with the values obtained in the **56**-plets. For other multiplets the determination of $c_{\mathcal{S}}$ is rather poor, such as in the $[\mathbf{56}, 4^+]$ resonance where only one strange baryon is known. Therefore, it is still possible that $c_{\mathcal{S}}$ has a similar value across multiplets, as one would expect. Finally, the c_4 coefficient, which plays no role in our analysis, turns out to have a large value and error from the fit to the $[\mathbf{56}, 2^+]$ multiplet. A careful consideration of the fit shows that the resonances $\Lambda(1820)$ and $\Sigma(2030)$ play an important role in determining the large value of c_4, while the fit gives a poor result for the mass of $\Lambda(1890)$. The chief difficulty in the $[\mathbf{56}, 2^+]$ multiplet is represented by the large value of c_4, or equivalently, the small masses of $\Lambda(1820)$ and $\Sigma(2030)$. It is somewhat puzzling that these are the only such states in the mass domain, which can be assigned to that multiplet. Although this point is not relevant for this work, it deserves to be studied more carefully.

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[2] One could make a similar plot to that in Fig. 1 using instead the values of c_1 suggested in Refs. [11,12,13,14,15,16]. As presented in Ref. [15], only one Regge trajectory is found in that case. However, the definition of the bases of operators differs from multiplet to multiplet in Refs. [11,12,13,14,15,16]. This is not the case in present study.

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References

[1] J. Carlson, J. Kogut and V.R. Pandharipande, Phys. Rev. D 27 (1983) 233 and references therein.
[2] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95 (2005) 261602.
[3] S. J. Brodsky and G. F. de Teramond, Phys. Rev. Lett. 96 (2006) 201601.
K. Nawa, H. Suganuma and T. Kojo, Phys. Rev. **D75** (2007) 086003.
[4] H. Forkel, M. Beyer, T. Frederico, arXiv:0705.4115; JHEP 0707 (2007) 77.
[5] C. Semay, F. Buisseret, N. Matagne and F. Stancu, Phys. Rev. D 75 (2007) 096001 [arXiv:hep-ph/0702075].
[6] A. Selem and F. Wilczek, arXiv:hep-ph/0602128.
[7] J. L. Gervais and B. Sakita, Phys. Rev. Lett. 52 (1984) 87; Phys. Rev. D 30 (1984) 1795.
[8] R. Dashen and A. V. Manohar, Phys. Lett. B 315 (1993) 425; ibid (1993) 438.
[9] D. Pirjol and C. L. Schat, Phys. Rev. D 67 (2003) 096009;
T. D. Cohen and R. F. Lebed, Phys. Rev. Lett. 91 (2003) 012001.
[10] R. Dashen, E. Jenkins, and A. V. Manohar, Phys. Rev. D 51 (1995) 3697.
[11] J. L. Goity, Phys. Lett. B 414 (1997) 140;
D. Pirjol and T. M. Yan, Phys. Rev. D 57 (1998) 1449; ibid. D 57 (1998) 5434;
C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Lett. B 438 (1998) 327; Phys. Rev. D 59 (1999) 114008.
[12] C. L. Schat, J. L. Goity and N. N. Scoccola, Phys. Rev. Lett. 88 (2002) 102002; J. L. Goity, C. L. Schat and N. N. Scoccola, Phys. Rev. D 66 (2002) 114014.
[13] J. L. Goity, C. L. Schat and N. N. Scoccola, Phys. Lett. B 564 (2003) 83.
[14] N. Matagne and Fl. Stancu, Phys. Rev. D 71 (2005) 014010.
[15] N. Matagne and Fl. Stancu, Phys. Lett. B 631 (2005) 7.
[16] N. Matagne and Fl. Stancu, Phys. Rev. D 74 (2006) 034014.
[17] J. L. Goity, *Large N_c QCD 2004*, World Scientific (2005) 211 [arXiv:hep-ph/0504101].
[18] C. E. Carlson and C. D. Carone, Phys. Lett. B 484 (2000) 260;
J. L. Goity, C. L. Schat and N. N. Scoccola, Phys. Rev. D 71 (2005) 034016;
J. L. Goity and N. N. Scoccola, Phys. Rev. D 72 (2005) 034024.
[19] C. E. Carlson and C. D. Carone, Phys. Rev. D 58 (1998) 053005; C. E. Carlson and C. D. Carone, Phys. Lett. B 441 (1998) 363;
J. L. Goity and N. N. Scoccola, Phys. Rev. Lett. 99 (2007) 062002.
[20] E. Witten, Nucl. Phys. B 160 (1979) 57
[21] N. Matagne and Fl. Stancu, Phys. Rev. D 73 (2006) 114025.
[22] Particle Data Group, W.-M. Yao et al., J. Phys. G 33 (2006) 1.
[23] E. Klempt, arXiv:nucl-ex/0203002.
[24] E. Jenkins and R. F. Lebed, Phys. Rev. D 52 (1995) 282.

Table 1
The coefficients c_1, c_{HF}, $c_{\mathcal{S}}$, c_4 (for the ground state only) and the theoretical masses (MeV) for the **56**-plets. The experimental masses used for the fit are also presented.

Multiplet	Baryon	Name, status	Exp. (MeV)	Theo (MeV)	c_1 (MeV)	c_{HF} (MeV)	$c_{\mathcal{S}}$ (MeV)	c_4 (MeV)	χ^2_{dof}
$[\mathbf{56}, 0^+]$	$\mathrm{N}_{1/2}$	N(939)****	939 ± 1	939 ± 2	362 ± 1	295 ± 3	208 ± 3	90 ± 5	9.1
	$\Lambda_{1/2}$	Λ(1116)****	1116 ± 1	1117 ± 1					
	${}^8\Sigma_{1/2}$	Σ(1193)****	1192 ± 4	1177 ± 4					
	${}^8\Xi_{1/2}$	Ξ(1318)****	1318 ± 3	1325 ± 4					
	$\Delta_{3/2}$	Δ(1232)****	1232 ± 1	1233 ± 2					
	${}^{10}\Sigma_{3/2}$	Σ(1385)****	1383 ± 3	1381 ± 1					
	${}^{10}\Xi_{3/2}$	Ξ(1530)****	1532 ± 1	1529 ± 2					
	$\Omega_{3/2}$	Ω(1672)****	1672 ± 2	1677 ± 2					
$[\mathbf{56}, 2^+]$	$\mathrm{N}_{3/2}$	N(1720)****	1700 ± 50	1682 ± 18	603 ± 5	767 ± 66	233 ± 46	416 ± 124	1.9
	$\Lambda_{3/2}$	Λ(1890)****	1880 ± 30	1822 ± 11					
	$\mathrm{N}_{5/2}$	N(1680)****	1683 ± 8	1682 ± 17					
	$\Lambda_{5/2}$	Λ(1820)****	1820 ± 5	1822 ± 11					
	${}^8\Sigma_{5/2}$	Σ(1915)****	1918 ± 18	1915 ± 38					
	$\Delta_{1/2}$	Δ(1910)****	1895 ± 25	1938 ± 18					
	$\Delta_{3/2}$	Δ(1920)***	1935 ± 35	1938 ± 18					
	$\Delta_{5/2}$	Δ(1905)****	1895 ± 25	1938 ± 18					
	$\Delta_{7/2}$	Δ(1950)****	1950 ± 10	1938 ± 18					
	${}^{10}\Sigma_{7/2}$	Σ(2030)****	2033 ± 8	2032 ± 18					
$[\mathbf{56}, 4^+]$	$\mathrm{N}_{9/2}$	N(2220)****	2245 ± 65	2245 ± 92	770 ± 20	398 ± 372	110 ± 94		0.13
	$\Lambda_{9/2}$	Λ(2350)***	2355 ± 15	2355 ± 21					
	$\Delta_{7/2}$	Δ(2390)*	2387 ± 88	2378 ± 84					
	$\Delta_{9/2}$	Δ(2300)*	2318 ± 132	2378 ± 84					
	$\Delta_{11/2}$	Δ(2420)*	2400 ± 100	2378 ± 84					
$[\mathbf{56}, 6^+]$	$\mathrm{N}_{13/2}$	N(2700)**	2806 ± 207	2806 ± 207	954 ± 40	342 ± 720			
	$\Delta_{15/2}$	Δ(2950)**	2920 ± 122	2920 ± 122					

Table 2
The coefficients c_1, $c_{\rm HF}$, $c_{\mathcal{S}}$ and the theoretical masses (MeV) for the **70**-plets. The experimental masses used for the fit are also presented.

Multiplet	Baryon	Name, status	Exp. (MeV)	Theo (MeV)	c_1 (MeV)	$c_{\rm HF}$ (MeV)	$c_{\mathcal{S}}$ (MeV)	$\chi^2_{\rm dof}$
$[\mathbf{70},1^-]$	$\mathrm{N}_{1/2}$	N(1535)****	1538 ± 18	1513 ± 14	529 ± 5	443 ± 19	148 ± 13	61
	${}^8\Lambda_{1/2}$	Λ(1670)****	1670 ± 10	1662 ± 6				
	$\mathrm{N}_{3/2}$	N(1520)****	1523 ± 8	1513 ± 14				
	${}^8\Lambda_{3/2}$	Λ(1690)****	1690 ± 5	1662 ± 6				
	${}^8\Sigma_{3/2}$	Σ(1670)****	1675 ± 10	1662 ± 6				
	${}^8\Xi_{3/2}$	Ξ(1820)***	1823 ± 5	1810 ± 15				
	$\mathrm{N}'_{1/2}$	N(1650)****	1660 ± 20	1661 ± 17				
	${}^8\Lambda'_{1/2}$	Λ(1800)***	1785 ± 65	1809 ± 12				
	${}^8\Sigma'_{1/2}$	Σ(1750)***	1765 ± 35	1809 ± 12				
	$\mathrm{N}'_{3/2}$	N(1700)***	1700 ± 50	1661 ± 17				
	$\mathrm{N}'_{5/2}$	N(1675)****	1678 ± 8	1661 ± 17				
	${}^8\Lambda'_{5/2}$	Λ(1830)****	1820 ± 10	1809 ± 12				
	${}^8\Sigma'_{5/2}$	Σ(1775)****	1775 ± 5	1809 ± 12				
	$\Delta_{1/2}$	Δ(1620)****	1645 ± 30	1661 ± 17				
	$\Delta_{3/2}$	Δ(1700)****	1720 ± 50	1661 ± 17				
	${}^1\Lambda_{1/2}$	Λ(1405)****	1407 ± 4	1514 ± 4				
	${}^1\Lambda_{3/2}$	Λ(1520)****	1520 ± 1	1514 ± 4				
$[\mathbf{70},2^+]$	$\mathrm{N}'_{1/2}$	N(2100)*	1926 ± 26	1987 ± 50	640 ± 16	400 (input)	120 ± 86	0.03
	$\mathrm{N}'_{5/2}$	N(2000)**	1981 ± 200	1987 ± 50				
	$\Lambda'_{5/2}$	Λ(2110)***	2112 ± 40	2108 ± 71				
	$\mathrm{N}'_{7/2}$	N(1990)**	2016 ± 104	1987 ± 50				
	$\Lambda'_{7/2}$	Λ(2020)*	2094 ± 78	2108 ± 71				
	$\Delta_{5/2}$	Δ(2000)**	1976 ± 237	1987 ± 50				
$[\mathbf{70},3^-]$	$\mathrm{N}_{5/2}$	N(2200)**	2057 ± 180	2153 ± 67	731 ± 17	249 ± 315	30 ± 159	0.15
	$\mathrm{N}_{7/2}$	N(2190)****	2160 ± 49	2153 ± 67				
	$\mathrm{N}'_{9/2}$	N(2250)****	2239 ± 76	2236 ± 81				
	$\Delta_{7/2}$	Δ(2200)*	2232 ± 87	2236 ± 81				
	${}^1\Lambda_{7/2}$	Λ(2100)****	2100 ± 20	2100 ± 28				
$[\mathbf{70},5^-]$	$\mathrm{N}_{11/2}$	N(2600)***	2638 ± 97		900 ± 20 (Est)			

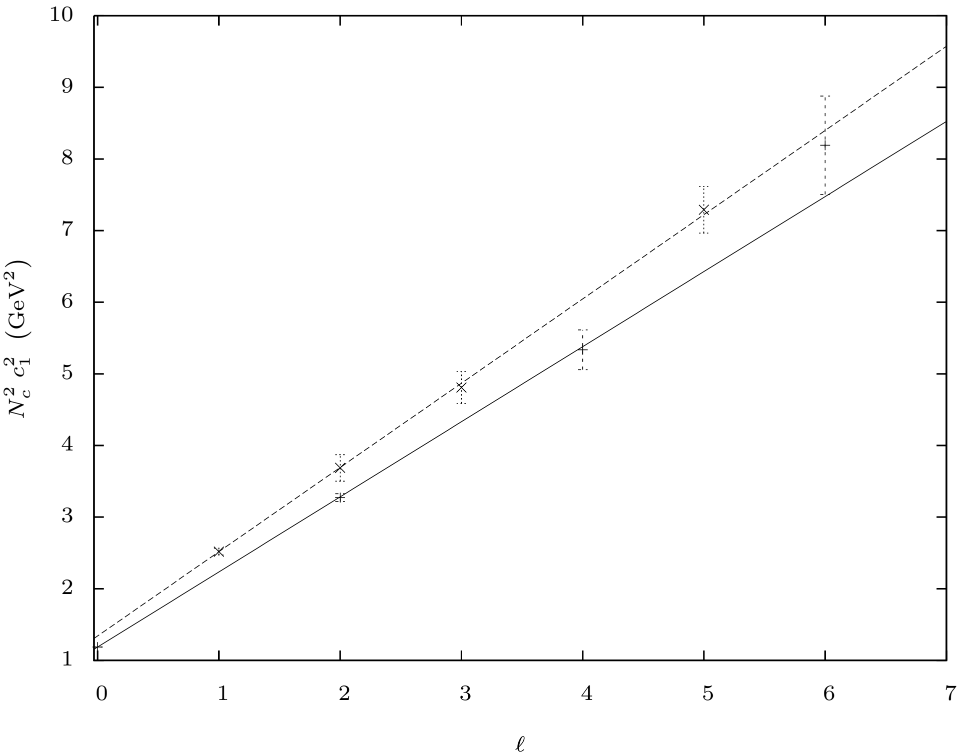

Fig. 1. Values of the coefficient $(N_c\ c_1)^2$ vs ℓ for the **56**-plets (+) and the **70**-plets ($\times$). The solid line represents the Regge trajectory for the symmetric multiplets and the dashed line, the Regge trajectory for the mixed symmetric multiplets.